模擬地盤による降雨浸透実験(2)－比抵抗による土中の水分量測定法－(英文)

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Experiments on Rain Infiltration in Soil (2)

——Mathematical development of specific resistance of mixed materials for measurement of soil-water content——

By

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Abstract

Soil-water content around the surface of the ground changes quickly in response to rainfall, because infiltrating water can easily be replaced by air existing among soil particles. Therefore, investigation for mechanisms of rainfall infiltration requires time-continuous or short periodic observation of the change of soil-water content in the ground. For this purpose, electrical methods are available. The relation between the specific resistance of mixed materials and the volume of each material is constructed by use of current density, on the hypothesis that on any closed surface the ratio of the area occupied by a material to the total area is uniform for every material.

The result is shown as,

$$\frac{1}{\rho}=\sum_{i=1}^{k}\frac{N_i}{\rho_i}, \qquad \sum_{i=1}^{k}N_i=1$$

where ρ is the apparent specific resistance, ρ_i is the specific resistance of the material i, N_i is the ratio of the area occupied by the material i to the total area of the closed surface. As the hypothesis is general, wide applications of above relation are possible.

Nomenclature

S	m^2	area of closed surface S
S_i	m^2	area of sectional surface
V	m^3	volume
N_i		ratio of S_i to S
I	A	total current
$\boldsymbol{J}$	$\mathrm{A/m^2}$	current density
$\boldsymbol{E}$	V/m	electric field intensity
$\boldsymbol{n}$		unit vector normal to the surface S
σ	$1/(\Omega\cdot\mathrm{m})$	specific conductance
ρ	$\Omega\cdot\mathrm{m}$	specific resistance
$\mathrm{d}s$	m^2	element of S

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$d\phi$	V	potential difference
$d\boldsymbol{r}$	m	element of pass

suffix i indicates the value of material of i

1 Preface

Natural disasters caused by rainfall, for example, landslide and slope failure, often occur after small rainfall. It is difficult to follow the mechanism of this kind of disaster caused by small rainfall. In many cases, velocity of infiltration of rainfall derived theoretically by coefficient of permeability is so small that it is difficult for water to reach slide surface in a short period. One idea which explains the above phenomena is that water which exists in soil before rainfall can move relatively quickly in response to rainfall. Hence, knowing of time-varying behaviour of water content in soil of every depth is very important in investigating the mechanism.

Measurement of water content in soil is, as mentioned above, very important but difficult. The reason for the difficulty lies in calibration processes. The method of direct measurement, in which wet soil is dried in oven and difference of soil weights before and after drying equals to the water content, has two demerits. First, it is not easy to obtain samples of soil from the ground from which water content is to be measured. Second, as a result of sampling, condition of the ground changes. So, in field or in laboratory, indirect methods are widely used. The methods of indirect measurement consist of calibration processes and transducers which generate measurable signals in response to the water content in soil. Consequently, the accuracy of the methods depends on the accuracy of calibration. In other words, the accuracy of the methods depends on the way in which the water content is measured, as well as the reproducibility of the transducers. For example, the reproducibility of the neutron water content meter is relatively better than other methods, tensiometer etc., but the accuracy of the method depends on calibration, in which the drying method is usually used.

From another point of view, transducers chosen should be fit for the purpose of measurement. Electric resistance method using nylon unit is used in the ground to detect somewhat qualitative change of water. The pore pressure meter method is used in agricultural field to measure water potential which indicates the motive force of water movement. Neutron water content meter is mainly used in the field of civil engineering to determine soil conditions after compaction.

The specific resistance method has been used in geotechnical survey. In electrical methods, there is no necessity for considering time lag of measurement and automatic measurement system can be made. In addition to these, in electrical methods, a period of one measurement is shorter than that of the other methods, such as neutron water content meter or tensiometer. This means that time-varving change of water content in soil can be followed by the measurement of specific resistance.

The relation between specific resistance and water content has been studied in various fields. Notable approaches have been made by Katsurayama (1957) and Yamashita (1971). Katsurayama constructed the apparent specific resistance which consists of three materials, soil particle, water and air.
He derived the equation as follows,

$$\frac{S}{\rho l}=\frac{S_1}{\rho_1 l}+\frac{S_2}{\rho_2 l}+\frac{S_3}{\rho_3 l}$$

where ρ is the apparent specific resistance, ρ_1, ρ_2, ρ_3 are the specific resistances of soil particle, water and air, respectively, S is the area through which electric current flows, S_1, S_2, S_3 are sectional areas occupied by soil particles, water and air, respectively, l is the length of these materials. Which means the apparent specific ıesistance is composed of occupied ratio and specific resistance of each component. He used above relation to measure water content in field by means of nylon units. Electric resistance (Ω), not specific resistance ($\Omega\cdot$m), was measured, according to his application. As the electric resistance can be affected by spatial location of electrodes, the merit of specific resistance was not evident in his application. Yamashita analysed specific resistance to measure void ratio of sedimentary rock, in which the void is filled by water. He constructed the apparent specific resistance of three materials, conductive rock, unconductive rock and void filled by water. The relation which he derived is similar to that which Katsurayama derived.

In the present paper, the specific resistance of mixed materials is generally derived. Exact proof is given for the relatio nwhich was derived by Katsurayama and Yamashita by the use of the current density. And the application of the specific resistance in measuring water content of soil becomes clear.

2 Hypothesis on mixed materials

In the papers mentioned above, soil models are given in geometrical form, for understanding the meaning of given formula of specific resistance of soil, although restrictions of application and modifications to the other soil models are not clear. In th issection, the hypothesis from which equation is derived is proposed for following discussions by the use of the electric current density.

Hypothesis I

On any closed surface S, in which a point source of current is included, any N_i, which is the ratio of the area of sectional surface S_i occupied by material i to the total area of the surface S, is uniform:
where

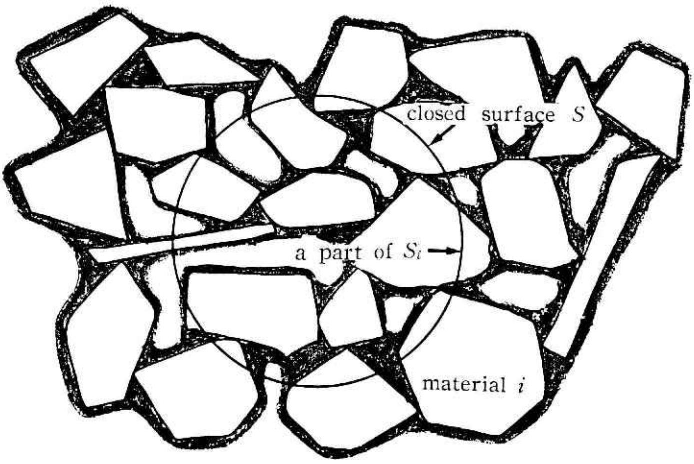
$$N_i=\frac{S_i}{S}$$

$$\sum_{i=1}^{k} S_i = S \quad \text{or} \quad \sum_{i=1}^{k} N_i = 1$$

i indicates material i,

k is the total number of materials.

Fig. 1 shows the closed surface.



Sectional area occupied by material i on S is totaled S_i. And the ratio $N_i = S_i/S$ is uniform on any closed surface S for each material.

Fig. 1 Closed surface in mixed materials.

3 Equation of specific resistance

The specific resistance is related to the current density and the electric field intensity, so the following discussion relates to the current density.

(a) Total current and current density

Current flows out the closed surface through the sectional surface occupied by each material. Therefore the total current I is written as,

$$I = \int_S \boldsymbol{J} \cdot \boldsymbol{n} \mathrm{d}s \tag{1}$$

where I is the total current (A), $\boldsymbol{J}$ is the apparent current density ($\mathrm{A \cdot m^{-2}}$), $\boldsymbol{n}$ is the unit vector normal to the surface S, and integration is performed on the closed surface S. Inner part of the integral is the component of current density normal to S. While, total current is the sum of each current which passes through material i.
Then,

$$I = \sum_{i=1}^{k} \int_{Si} \boldsymbol{J}_i \cdot \boldsymbol{n} \mathrm{d}s_i \tag{2}$$

where $\boldsymbol{J}_i$ is the current density of material i and S_i is the sectional surface occupied by material i. So $\mathrm{d}s_i$ and $\mathrm{d}s$ are connected by N_i,

$$\mathrm{d}s_i = N_i \mathrm{d}s_i \tag{3}$$

Therefore,

$$\int_{S_i} \boldsymbol{J}_i \cdot \boldsymbol{n} \, \mathrm{d}s_i = \int_{S} \boldsymbol{J}_i \cdot \boldsymbol{n} \; N_i \mathrm{d}s \qquad (4)$$

Eq. 2 becomes,

$$I = \sum_{i=1}^{k} \int_{S} (\boldsymbol{J}_i N_i) \cdot \boldsymbol{n} \; \mathrm{d}s \qquad (5)$$

Equating the inner part of **Eq. 1** and that of **Eq. 5,** following relation is obtained,

$$\boldsymbol{J} = \sum_{i=1}^{k} \boldsymbol{J}_i \; N_i \qquad (6)$$

Eq. 6 means that the apparent current density is composed of $\boldsymbol{J}_i$ and N_i.
Fig. 2 shows the above relations.

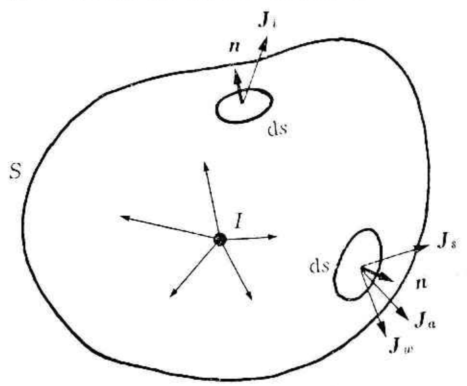


Fig. 2 The total current I and the current density $\boldsymbol{J}_i$ of material i on the closed surface S.

$\boldsymbol{J}_i$ is the current density which is defined in the material i. The current of $\boldsymbol{J}_i$ through sectional surface ds is $\boldsymbol{J}_i N_i$ds. Therefore the vertical component of the current to the surface ds is $\boldsymbol{J}_i N_i \mathrm{d}s \cdot \boldsymbol{n}$. If the mixture consists of soil particles, air and water, the total current through surface ds is composed of the current of each material. So the vertical component of the total current on ds is $(\boldsymbol{J}_s N_s + \boldsymbol{J}_a N_a + \boldsymbol{J}_w \boldsymbol{n}_w) \cdot \boldsymbol{n}\mathrm{d}s$. Suffixes s, a, w mean soil particle, air ane water, respectively.

(b) Potential difference

Potential difference dϕ along the pass d$\boldsymbol{r}$ is defined by,

$$\mathrm{d}\phi = \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r} \qquad (7)$$

where $\boldsymbol{E}$ is an electric field intensity $(\mathrm{V \cdot m^{-1}})$, and is replaced by current density $\boldsymbol{J}$ with the relation of,

$$\boldsymbol{J} = \sigma \boldsymbol{E} = \frac{1}{\rho} \boldsymbol{E} \qquad (8)$$

where σ is electric conductivity or specific conductance $(\Omega^{-1} \cdot \mathrm{m}^{-1})$, and ρ is resistivity or specific resistance $(\Omega \cdot \mathrm{m})$. Hence d$\phi$ is rewritten as,

$$\mathrm{d}\phi = \rho \boldsymbol{J} \cdot \mathrm{dr} \qquad (9)$$

The mixture under discussion consists of more than two materials. Then the specific resistance ρ and the current density $\boldsymbol{J}$ in **Eq. 9** mean apparent value.

(c) Average specific resistance

According to hypothesis I, current passes through each material i for any pass d$\boldsymbol{r}$. Hence potential difference dϕ is expressed by the specific resistance and the current

density of material i.
So,

$$\mathrm{d}\phi = \rho_i \boldsymbol{J}_i \cdot \mathrm{d}\boldsymbol{r} \tag{10}$$

Where ρ_i is the specific resistance of material i.
Substituting **Eq. 10** to **Eq. 9**, the following relation is obtained,

$$\boldsymbol{J}_i = \frac{\rho}{\rho_i}\boldsymbol{J} \tag{11}$$

where $\boldsymbol{J}$ is the apparent current density used in **Eq. 1.** Then **Eq. 6** becomes,

$$\boldsymbol{J} = \sum_{i=1}^{k} \boldsymbol{J}_i N_i = \rho \boldsymbol{J} \sum_{i=1}^{k} \frac{N_i}{\rho_i} \tag{12}$$

Finally,

$$\frac{1}{\rho} = \sum_{i=1}^{k} \frac{N_i}{\rho_i} \tag{13}$$

or

$$\sigma = \sum_{i=1}^{k} \sigma_i N_i \tag{14}$$

where ρ (σ) is the apparent specific resistance (conductance), ρ_i (σ_i) is the specific resistance (conductance) of material i, N_i is the ratio of area defined in the hypothesis I. **Eq. 14** shows the apparent electric conductivity, which is the inverse value of the apparent specific resistance, equal to the sum of the electric conductivity of each component weighted by N_i.
Substituting **Eq. 6** and **Eq. 13** into **Eq. 9** the potential difference becomes,

$$\mathrm{d}\phi = \frac{\sum_{i=1}^{k} \boldsymbol{J}_i N_i}{\sum_{i=1}^{k} \frac{N_i}{\rho_i}} \cdot \mathrm{d}\boldsymbol{r} \tag{15}$$

4 Application for measuring water content in soil

The water content can be related to the specific resistance by the use of N_i which is defined in the hypothesis I.

(a) Water content

The volume of material i in any closed surfacs S can be derived by the use of N_i according to the hypothesis I. If the volume of material i is denoted by V_i,

$$V_i = \int_{r_1}^{r_2} N_i S(r)\mathrm{d}r = N_i \int_{r_1}^{r_2} S(r)\mathrm{d}r = N_i V \tag{16}$$

where $S(r)$ is the sectional area of the volume of S perpendicular to the axis r, r_1 and r_2 are the boundary points of S along the axis of r.
From the hypothesis I, $N_i S(r)$ is the sectional area occupied by material i. **Fig. 3**

shows the meaning of **Eq. 16.**
Then,

$$N_i=\frac{V_i}{V} \tag{17}$$

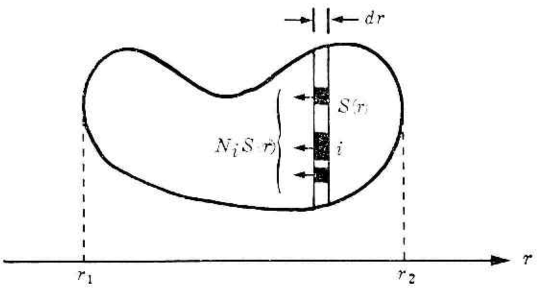


Fig. 3 Volume of material i.

$S(r)$ is sectional area perpendicular to the axis of r. From the hypothesis I, $N_iS(r)\mathrm{d}r$ is the volume element of material i corresponding to the element of dr.

Eq. 17 means that the N_i equals to the ratio of volume of material i to the total volume of S in any closed surfac S. If the material i is water, N_i means the ratio of volume of water to the total volume of S.

(b) Sandy soil

Particle sizes of sandy soil are relatively homogeneous, and humic zone may not be developed on the surface of particles. Hence there can be three materials, mineral, air and water. **Eq. 13** is rewritten explicitly for sandy soil,

$$\frac{1}{\rho}=\frac{N_s}{\rho_s}+\frac{N_a}{\rho_a}+\frac{N_w}{\rho_w} \tag{18}$$

where suffixes s, a and w indicate mineral, air and water.
The value of specific resistance of mineral and air are larger than that of water,

$$\rho_w \ll \rho_s,\ \rho_a \tag{19}$$

Then **Eq. 18** reduces to,

$$\frac{1}{\rho}\approx\frac{N_w}{\rho_w} \tag{20}$$

It means that the apparent specific resistance nearly equals the value of the specific resistance of water devided by the ratio of area N_i.

According to **Eq. 17,** the water content is expressed by the use of specific resistance. Finally,

$$N_w=\frac{V_w}{V}\approx\frac{\rho_w}{\rho} \tag{21}$$

It means that the water content N_w, which means the ratio of the total volume of water V_w to the total volume of soil V which includes the volume of water, air and soil particles, is the ratio of the specific resistance of water to the apparent specific resistance of soil.

(c) Humic soil

In humic zone or in void among fine particles, there exist unmovable water. As conductive materials are soluted in this kind of water, the specific resistance of unmovable water is lower than that of movable water. In this case, four materials must be considered, mineral, air, movable water and unmovable water. **Eq. 13** is rewritten as,

$$\frac{1}{\rho}=\frac{N_s}{\rho_s}+\frac{N_a}{\rho_a}+\frac{N_m}{\rho_m}+\frac{N_u}{\rho_u} \tag{22}$$

where suffix s, a, m and u indicate mineral, air, movable and unmovable water, As in the case of sandy soil, the value of specific resistance of mineral and air are larger than those of these kind of water.

$$\rho_m,\ \rho_u \ll \rho_s,\ \rho_a \tag{23}$$

Eq. 22 becomes,

$$\frac{1}{\rho}\approx\frac{N_m}{\rho_m}+\frac{N_u}{\rho_u} \tag{24}$$

5 Concluding remarks

The specific resistance (Ω·m) is another quantity of the electric resistance (Ω). The electric resistance is composed of the specific resistance and the geometrical location of the electrodes. So, the specific resistance depends only on the water content, but the electric resistance varies under the influence of the water content and the location of the electrodes. Therefore, it is beneficial to measure the specific resistance directly in applications. Application for dried soil needs a somewhet extended modification introducing the surface resistivity of soil particles.

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模擬地盤による降雨浸透実験(2)
—比抵抗による土中の水分量測定法—

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降雨の浸透による土中水分の変動は，地表付近では浸透水と地盤中の空気が比較的スムーズに置き換わるのでその変動も早く現われる．このような土中水分の変化をとらえるには連続的あるいは十分短い間隔の観測を行わなければならない．それには電気的な方法がすぐれている．本報告では土中の任意の体積中に含まれる各物質の割合が一定であるということを含む仮定に基づき，混合物質の比抵抗とその体積との関係を導出した．結果は次式である．

$$\frac{1}{\rho}=\sum_{i=1}^{k}\frac{N_i}{\rho_i}, \qquad 但し \qquad \sum_{i=1}^{k}N_i=1$$

ここで ρ は見掛けの比抵抗，ρ_i は物質 i の比抵抗，N_i は物質 i の体積と全体積の比に相当する．上述した仮定は一般的であるので，土中の直方体内部での抵抗理論の応用である多孔質ブロック内での抵抗測定法と比較すれば，理論式の中に土の特徴を表現することが容易になり，また測定手段として4電極法などあらゆる比抵抗測定法が使用できることが示されている．